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HEAT TRANSFER TO NEWTONIAN FLUIDS IN COILED PIPES IN LAMINAR FLOW

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Abstract—Data on pressure drop and heat transfer to aqueous solutions of glycerol flowing in different types of coiled pipes are presented for laminar flow in the range of N_{Re} from 80 to 6000. An empirical correlation is set up which can account the present data as well as the data available in literature within ± 10 per cent deviation. Conventional momentum and heat transfer analogy equation is used to analyse the present data.

NOMENCLATURE

C_p, specific heat at constant pressure, cal/g degC;

D_c, diameter of the helical coil, cm; suffix AVG indicates the average diameter of the spiral coil;

D_i, inner diameter of the tube, cm;

D_i/D_c, curvature ratio of the helical coil;

$D_i/(D_c)_{AVG}$, average curvature ratio of the spiral coil;

f_c, Fanning's friction factor for helical coils and spiral coils given by $\Delta H = 2f_c L V^2/g_c D_i$;

g_c, acceleration due to gravity, cm/s^2;

ΔH, head loss, cm of liquid;

h_a, heat-transfer coefficient evaluated at arithmetic mean temperature difference, viz. $[(t_w - t_1) + (t_w - t_2)]/2$;

i, intercept;

k_b, thermal conductivity at bulk temperature, Kcal/mh degC;

L, length of the coil, cm;

N_{Dean}, Dean's number $[= N_{Re}\sqrt{(D_i/D_c)}]$;

N_{Gz}, Graetz number $[= WC_p/K_bL]$;

N_{Nu}, Nusselt number $[= h_a D_i/K_b]$;

N_{Pr}, Prandtl number $[= C_p\mu/K_b]$;

N_{Re}, Reynolds number $[= D_i V\rho/\mu]$; suffix Cr indicates critical Reynolds number;

t, temperature, °C; suffixes w, b, 1, 2 indicate wall, bulk, initial and final temperatures, respectively;

v, mean velocity, cm/s;

W, weight rate of flow, g/s;

ρ, density, g/cm^3;

μ, viscosity, cp.

INTRODUCTION

THERE ARE different types of coiled pipes, of which attention has been focused on helical and spiral tube coils in the present studies. The design procedure for helical coils has been presented by Shrprintz [1], whereas for Archimedian spiral tube coils the present authors [2] have extended the method proposed for the construction of spiral curve by Grazda, to include the number of turns for a given length. Archimedian spiral coils are used as pancakes in industries. These coils find extensive use as heat exchangers.

Helical coils are of constant curvature ratios whereas in spirals the curvature ratio varies along the length according to a fixed law.

The phenomena of momentum and heat transfer in coiled pipes have not been fully understood though there are various references available in literature.

The first theoretical study in curved pipes has been made by Dean [3]. White [4] has proposed an empirical formula to calculate the friction factors for helical coils, incorporating Dean's number as,

$$\frac{f_c}{f_s} = \{1 - [1 - (11{\cdot}6/N_{\mathrm{Dean}})^{0{\cdot}45}]^{2{\cdot}2}\} - 1$$

for $11{\cdot}6 < N_{\mathrm{Dean}} < 2000$; $f_s = 16/N_{Re}$.
Recently Mori and Nakayama [5] have found that Dean's solution is applicable to quite low Dean number regions and Dean's formula for resistance coefficient is not valid.

Kubair and Varrier [6] have conducted pressure drop measurements in helical coils and found that their results agreed with White's correlation at lower ranges of Reynolds numbers. In the neighbourhood of transition region, deviations are observed.

Very little work on pressure drop in spirals has been reported in the literature. Noble *et al.* [7] have presented the results of pressure drop for water and oil flowing in a modified 8XF-12 heliflow, a standard unit manufactured by the Graham Manufacturing Company, U.S.A.

The main flow in coiled pipes is accompanied by secondary flow. The free vortex flow occurring in coiled pipes makes the velocity as well as temperature profiles asymmetric. The distortion of the profile is due to the transfer of momentum in radial direction, by virtue of centrifugal forces. The onset of turbulence in coiled pipes is delayed owing to the suppression of transverse motions of fluid elements by centrifugal forces. The criterion for estimating the onset of turbulence similar to the one proposed by Ito [8], viz.

$$(N_{Re})_{Cr} = 1{\cdot}273 \times 10^4 \times (D_i/D_c)^{0{\cdot}2}$$

has been suggested for helical coils by Kubair and Varrier [6] and extended to spiral tube coils by Kubair and Kuloor [9] by replacing the curvature ratio term by the average curvature ratio of the spiral tube coil.

Depending on the relative contribution of centrifugal forces, the friction factors for coiled pipes differ from the corresponding values in straight pipes. Under double helical streamline flow condition, the difference in friction factors for coiled and straight pipes is more appreciable and progressively decreases in turbulent flow. The intensity of free vortex flow has been correlated with the geometrical parameters by Yarnell and Ngler [10] for pipe bends. Kubair and Kuloor [11, 12] have found that the intensity of free vortex flow depends on the length and curvature ratio of coiled pipes.

Berg and Bonilla [13] have reported heat-transfer data on oils flowing in helical coils under laminar flow conditions, at constant tube wall temperature conditions. They have found that by coiling the straight pipe, much improvement in heat transfer with respect to pressure drop cannot be obtained. Higher heat-transfer coefficients for coiled pipes at higher Graetz numbers have been reported by Berg and Bonilla [13] for helical coils, and by Noble *et al.* [7] for spiral coils.

Seban and McLaughlin [14] have reported higher heat-transfer coefficients for helical coils in laminar flow under constant heat flux conditions even at very low Reynolds numbers.

Hence one can find the difference in the trend of results obtained by different investigators.

The present work is taken up to collect heat-transfer data in different types of coiled pipes under constant tube wall temperature conditions and correlate the results in laminar flow.

The range of variables covered and the different types of coiled pipes used in the present study are given in Table 1.

Table 1

Helical or spiral coil	D_i/D_c or $D_i/(D_c)_{\mathrm{AVG}}$	D_i (cm)	No. of turns	Range of N_{Re}	Length (cm)	Range of N_{Gz}
H_{I}	0·037	0·6549	7	150–5100	463·9	6–150
H_{II}	0·056	0·6407	12	60–4950	468·1	3–144
H_{III}	0·074	0·954	10	170–3700	489·1	10–195
H_{IV}	0·097	1·27	10	200–4700	475·1	15–223
S_{II}	0·022	0·642	4·5	200–6000	486·4	11–160
S_{III}	0·031	0·9466	5·0	170–4000	484·6	9–180

62% and 10% glycerol solutions are used to cover the ranges for H_{I}, H_{III}, H_{IV}, S_{II} and S_{III} whereas 70% and 43·5% glycerol solutions are used in the case of H_{II}.

EXPERIMENTAL

The arrangement of apparatus used in the present studies is given in Fig. 1. Except the test section, the arrangement is the same for spiral coils. The test section for spirals is shown in Fig. 2.

The test liquid is pumped from the storage tank A to the overhead tank B and then through the rotameter R. For lower flow rates, the liquids are pumped from the overhead tank directly. The valves V_1 and V_2 are used to adjust the flow rate. The valve V_3 regulates the position of the float in the rotameter. E is a straight thermal entry length of copper pipe, more than 100 pipe diameters. S is a steam chest, in which helical coil C is placed in horizontal position. G_1 and G_2 are glands, which are provided with rubber packings, so that when copper tube is inserted, it should not make metal to metal contact. M_1, M_2 and M_3 are the manometer leads taken from static probes P_1, P_2 and P_3 on coil C. M is a mixing cup in which the spiral bundle of wire is placed to promote effective mixing. The inlet and outlet temperatures of the test liquid, and steam temperature are recorded using thermometers of accuracy 0·1 degC. A steam trap T_r is attached to the steam chest to remove the condensate continuously. Th_1, Th_2 and Th_3 are 28 gauge copper–constantan thermocouple wires, used to measure the tube wall temperature, by measuring the thermo-e.m.f. using a Kaycee type potentiometer. F is a metallic flange, connected to the steam chest C, using a rubber gasket. The apparatus is lagged with asbestos–magnesia powder mixture, to prevent heat losses.

The test section for spirals is presented in Fig. 2. It consists of a copper steam jacket J, to which are brazed two glands G_1 and G_2. A steam trap T_r is attached through a valve V_1. To the gland G_2, a mixing cup is joined by another gland connection, using asbestos rope, to avoid metal to metal contact. The flange F consists of eight small brass glands through which

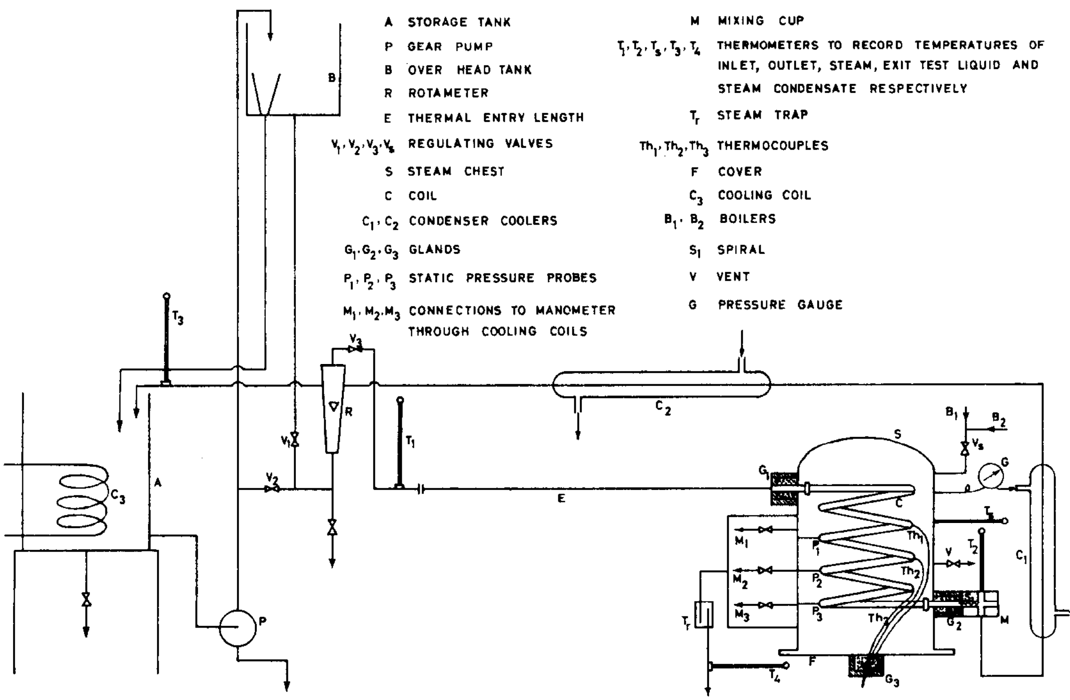

FIG. 1. Arrangement of apparatus for studies in heat transfer in helical coils.

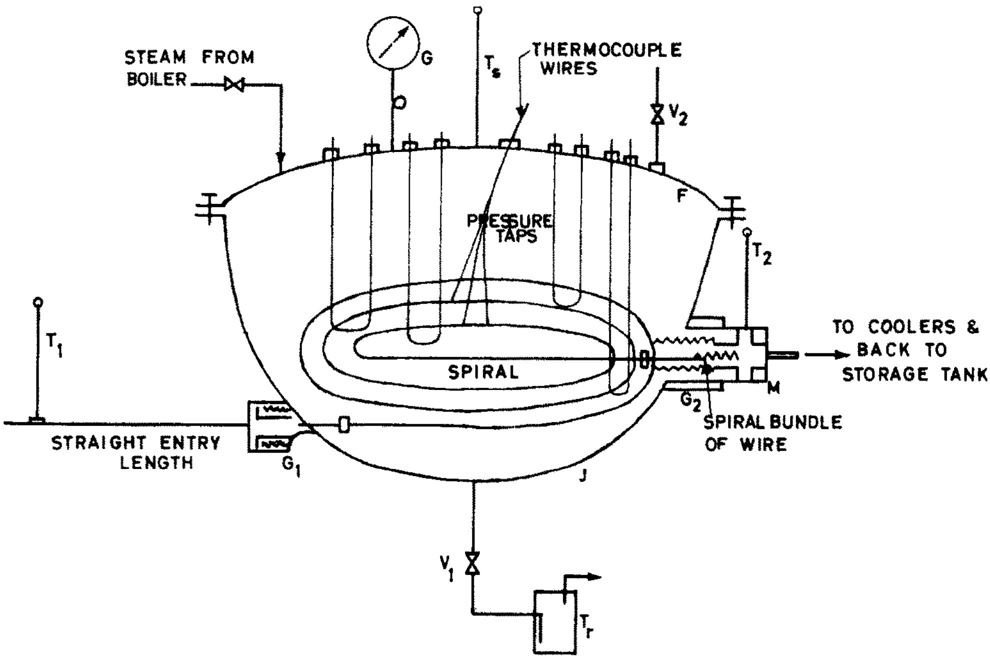

FIG. 2. Spiral coil heat exchanger.

connections of static pressure probes can be taken outside the test section. Pressure gauge G, vent V_2, thermometer T_s, thermocouples Th_1, Th_2, Th_3 are taken out through other glands. Connections from the boiler are provided in the flange F. The flange is fixed to the steam jacket J, in which spiral coil S is placed. The steam jacket is completely lagged with a mixture of asbestos–magnesia powder.

METHOD OF OPERATION

Boilers B_1 and B_2 are filled with water up to a level and heaters are switched on. When the pressure of the steam is about 2 psig, the liquid is pumped through the helical coil C or spiral coil S. The level of the float in the rotameter is adjusted at a particular value. When the pressure of the steam is about 5 psig, the valve V_S is opened and the temperature of the steam is recorded. The non-condensable gases are removed through the vent V. When the temperature of steam is about 96·6°C, the simmerstats attached to the heaters are adjusted, such that the steam temperature is maintained at a predetermined value. Water is circulated through the condensers C_1, C_2 and the cooling coil C_3 Then the steam temperature decreases but it is maintained constant by adjusting the simmerstats. The temperatures of the liquid are noted by the thermometers T_1, T_2 and T_3. T_3 is an ordinary thermometer of 1 degC accuracy used to record the temperature of the test liquid, returning from the test section to the storage tank. During the experiments, the density of the test liquid is maintained constant, by keeping the concentration of the test liquid constant by adding water or glycerol. The density of the test liquid is noted by a hydrometer. The condensate is removed through the steam trap T_r and collected in a bucket. The temperature of the condensate is noted by the thermometer T_4 to avoid the subcooling of the condensate.

For a particular flow rate, when steady state conditions are attained, the weight rates of the liquid and of condensate are found out. After checking the heat balance, the readings of inlet, outlet temperature of the liquid, steam temperature and weight rates of the liquid are entered as final values. Only when the heat balance is more than 95 per cent are the readings accepted.

The thermo-e.m.f. produced is noted by the Kaycee type potentiometer.

RESULTS AND DISCUSSION

Pressure drop

The results of pressure drop (isothermal) for helical and spiral tube coils are presented in Figs. 3, 4 and 5. The friction factors are calculated using the equation

$$\Delta H = 2f_c L V^2 / g_c D_i$$

and μ, viscosity, is evaluated at the liquid temperature for isothermal conditions and at film temperature for non-isothermal conditions.

Data on pressure drop for helical coils are agreeing with White's [4] correlation as shown in Fig. 6.

Heat transfer

The intensity of free vortex flow in coiled pipes is found to change with the length and the curvature ratio of the coiled pipes by Kubair and Kuloor [11, 12].

Prabhudesai and Shah [15] have reported different heat-transfer coefficient for a coil, with different number of turns.

Seban and McLaughlin's [14] data are recomputed in terms of Graetz number and Nusselt number, for their smaller coil of curvature ratio 0·056. Data have been reported by Berg and Bonilla [13] on a coil of same curvature ratio. The trend of the data obtained by Berg and Bonilla differs entirely from that obtained by Seban and McLaughlin since higher heat-transfer rates are obtained at lower Graetz numbers according to Seban and McLaughlin and *vice versa* according to Berg and Bonilla. The different boundary conditions at the tube wall may have an important role on the trend of results obtained by different investigators. Seban and McLaughlin have reported heat-transfer data at constant heat flux conditions whereas Berg and Bonilla have reported at constant tube wall temperature conditions. Hence the results are not uniform.

Since the resistance to momentum transfer is governed by the length and the curvature ratio of the coiled pipes [11, 12], the resistance to heat transfer is also governed by the same variables. Hence the heat-transfer coefficients are evaluated in the present studies at arithmetic mean temperature difference and are correlated

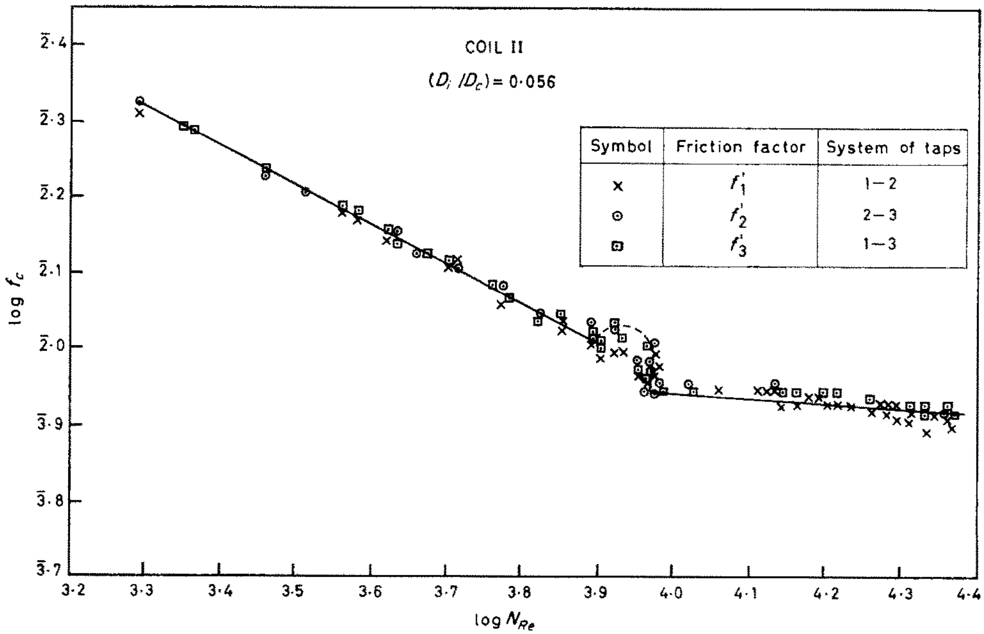

FIG. 3. Plot of $\log f_c$ vs. $\log N_{Re}$.

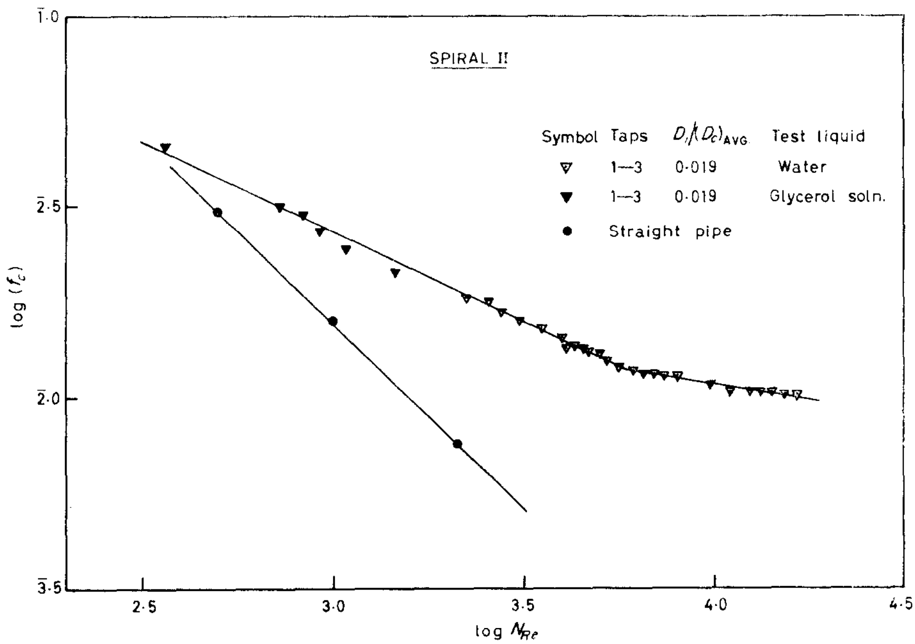

FIG. 4. Plot of $\log f_c$ vs. $\log N_{Re}$.

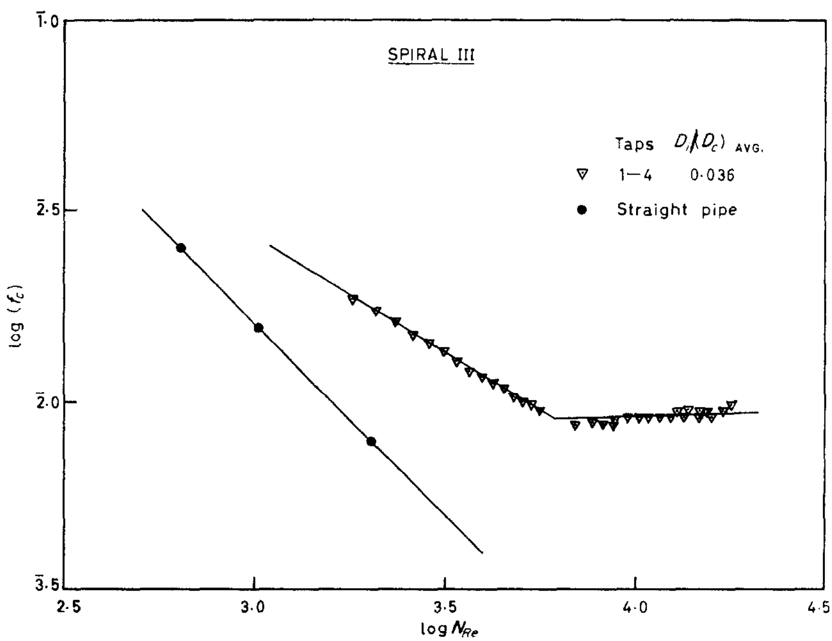

FIG. 5. Plot of $\log f_c$ vs. $\log N_{Re}$.

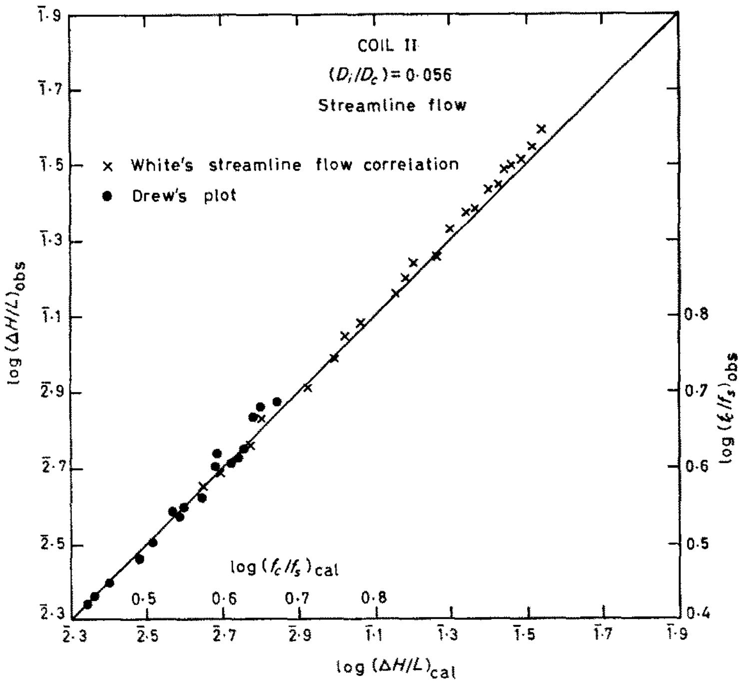

FIG. 6. Comparison of the present data with the existing correlations.

using Nusselt and Graetz numbers with the geometrical parameters of the coiled pipes. The physical properties of the test liquid are evaluated at bulk temperature. The thermal conductivity and specific heats are computed using Kern's empirical rule [16]. The viscosity of the test liquid is determined at various temperatures by using Höppler's viscometer and interpolated at intermediate temperatures from the calibration curve.

The results of heat transfer are presented in Figs. 7–12. The present data on heat transfer in helical coil II coincide with Seban and McLaughlin's data [14] at lower Graetz numbers and deviate at higher Graetz numbers. Comparison of the data obtained in helical coil II with Seban and McLaughlin's data on their smaller coil reveals that L/D_t ratios and range of Prandtl numbers have significant effect on heat-transfer rates, although the curvature ratios of the helical coils are the same. Hence it is justified to correlate the results using Graetz number.

Higher heat-transfer coefficients than obtained in straight pipes are possible for all the coils. The equation

$$N_{Nu} = 1{\cdot}75(N_{Gz})^{1/3}$$

which is an approximation of Graetz theoretical solution [17] for laminar flow heat transfer for the case of uniform wall temperature and parabolic distribution of mass velocity for values $N_{Gz} > 10$, is used, to compute heat-transfer coefficients for straight pipes.

Higher heat-transfer rates for coils than those in straight pipes have been reported by Hawes [18] and Aranov [19].

In the light of this information, the data are analysed as follows. The average of the slopes of the lines in Figs. 7–12 is determined to fix the index of Graetz Number. Then the intercepts i are calculated for all the coils and correlated with the curvature ratio of helical coils or average curvature ratio of spiral coils as,

$$i = 1{\cdot}98 + 1{\cdot}8(D_t/D_c).$$

The agreement between the observed and

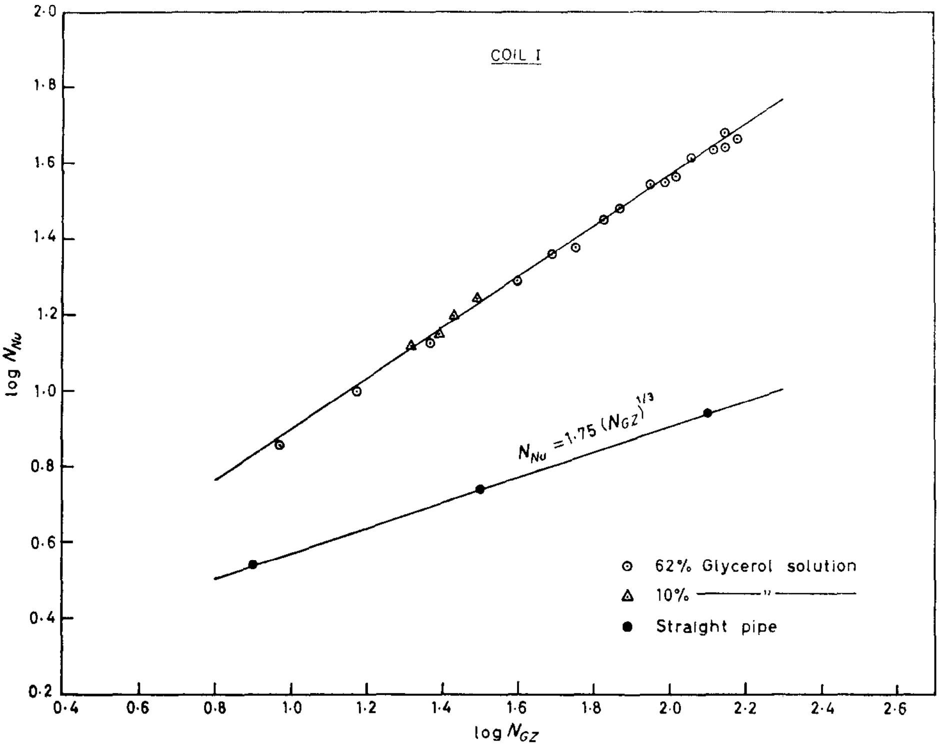

FIG. 7. Plot of log N_{Nu} vs. log N_{Gz}.

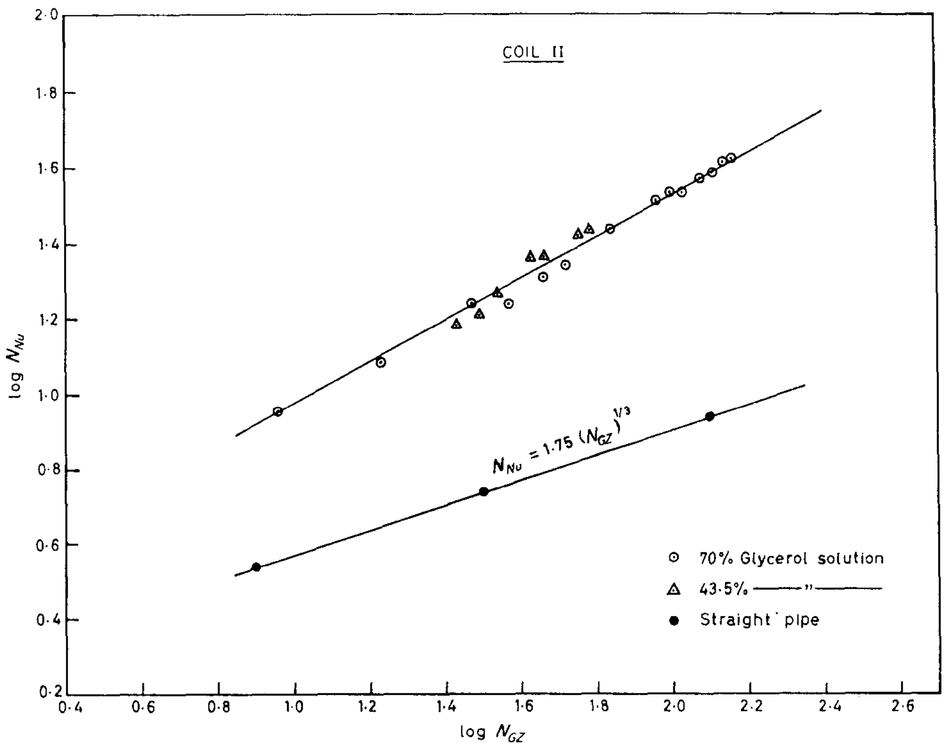

FIG. 8. Plot of log N_{Nu} vs. log N_{Gz}.

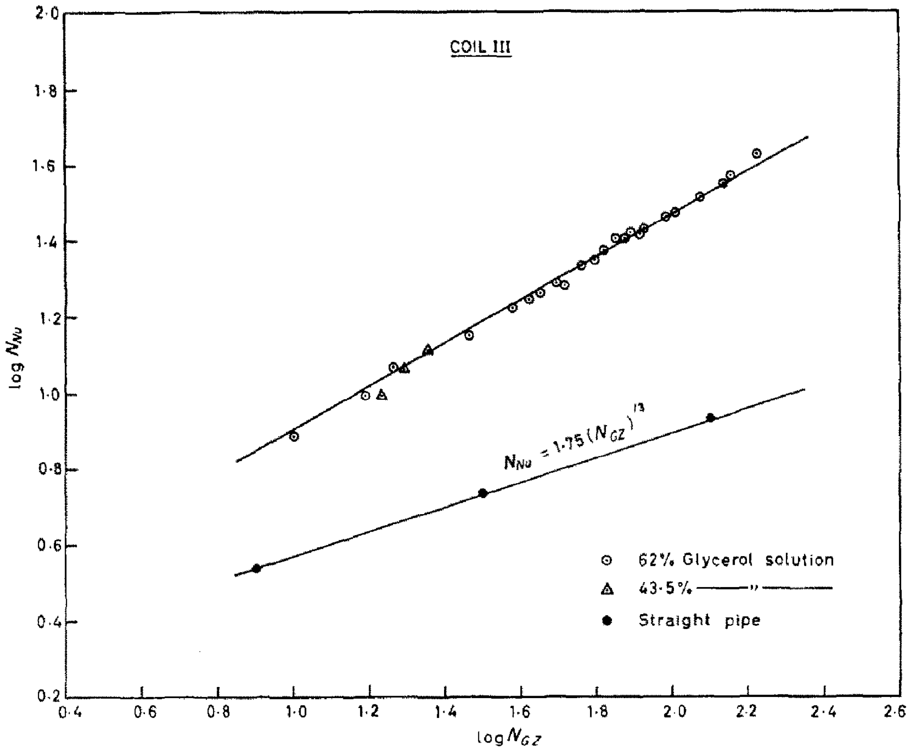

FIG. 9. Plot of log N_{Nu} vs. log N_{Gz}.

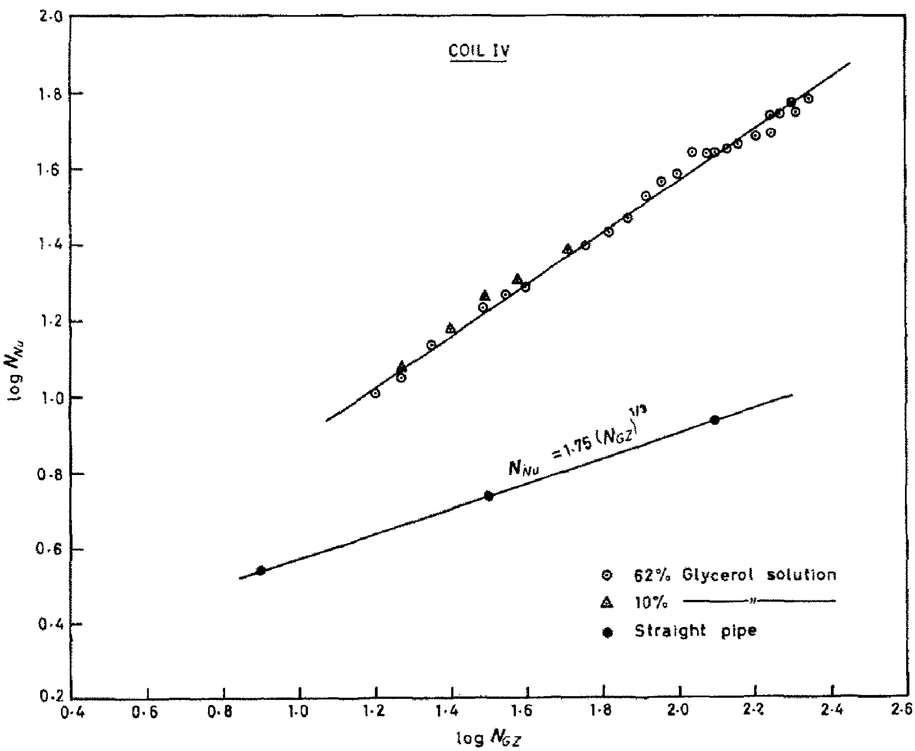

FIG. 10. Plot of log N_{Nu} vs. log N_{Gz}.

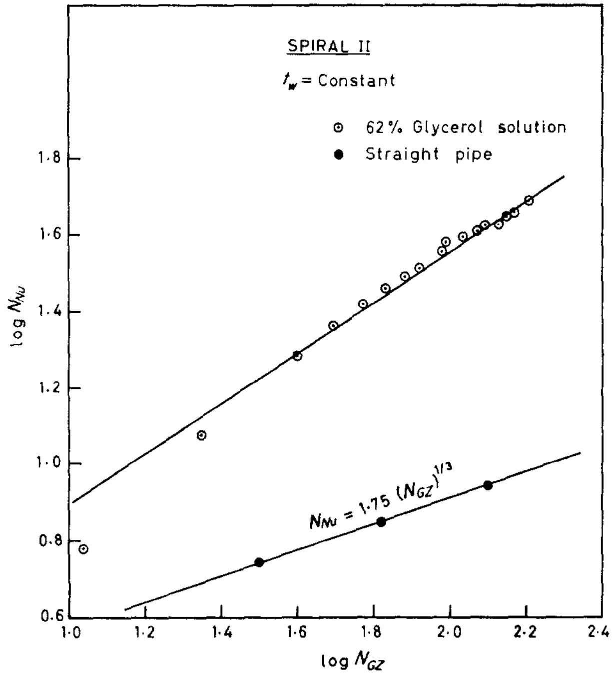

FIG. 11. Plot of log N_{Nu} vs. log N_{Gz}.

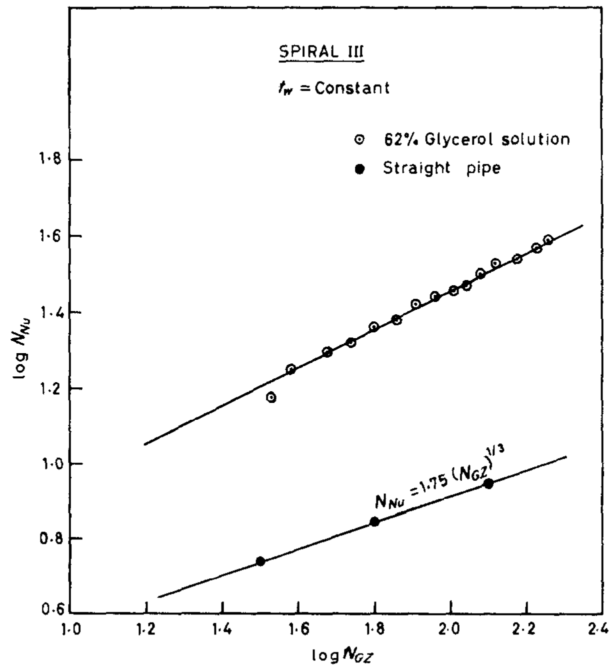

FIG. 12. Plot of log N_{Nu} vs. log N_{Gz}.

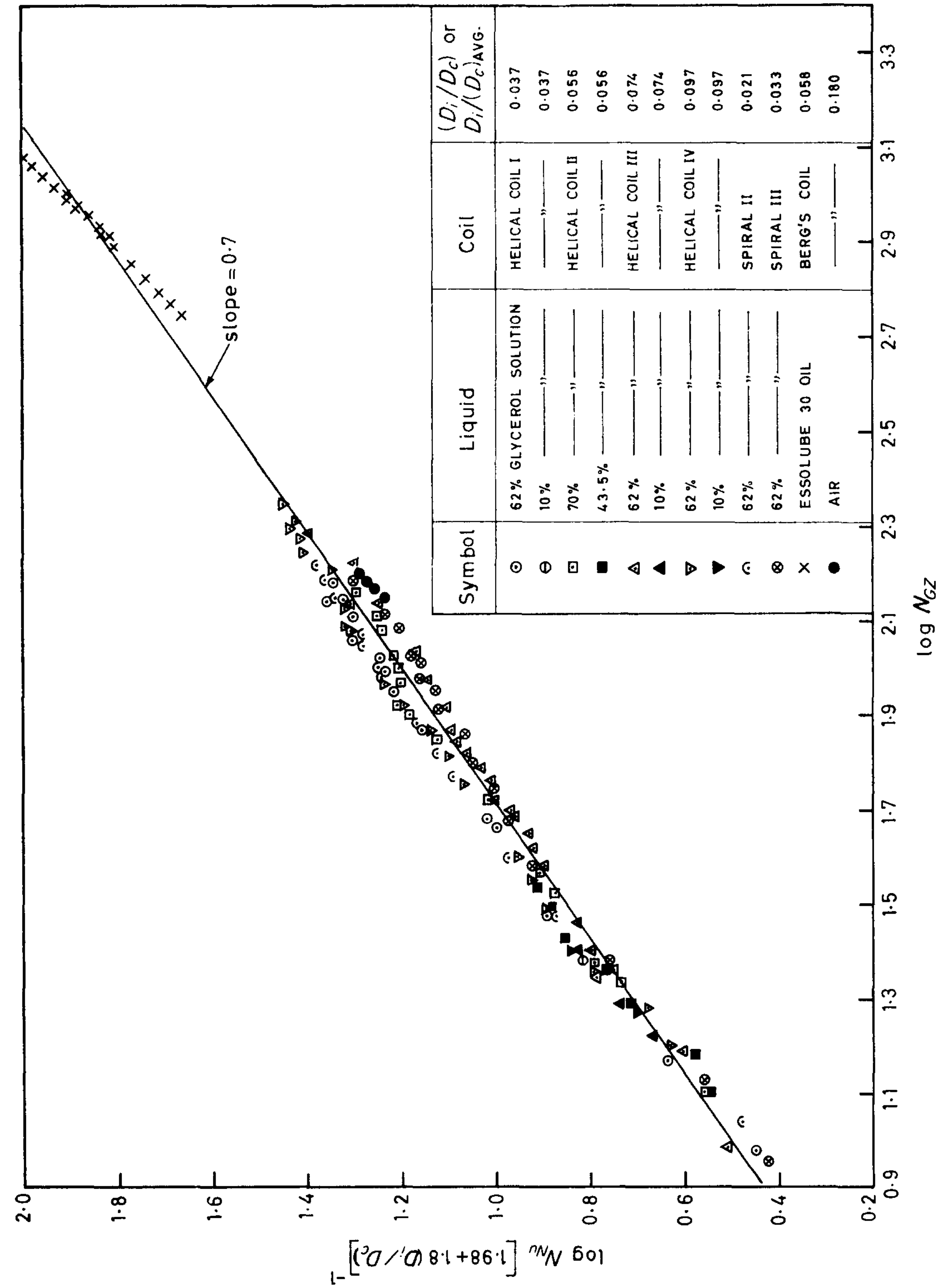

FIG. 13. Plot of log $N_{Nu}[1{\cdot}98 + 1{\cdot}8(D_i + D_c)]^{-1}$ vs. log N_{Gz}.

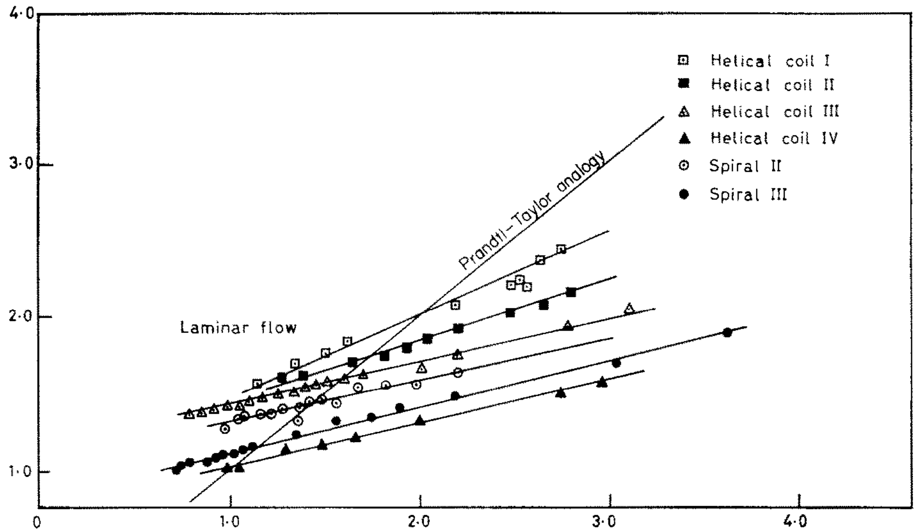

FIG. 14. Plot of $\frac{2f_cL}{D_i}\frac{1}{1+5(f_c/2)^{\frac{1}{2}}(N_{Pr}-1)}$ vs. $2{\cdot}303 \log \frac{t_w - t_1}{t_w - t_2}$.

calculated values can be found from Fig. 13 to be within a maximum deviation of ± 10 per cent and average deviation less than 5 per cent. The correlation can be written as,

$$N_{Nu} = [1{\cdot}98 + 1{\cdot}8(D_i/D_c)]N_{Gz}^{0{\cdot}7}$$

and holds good for $10 < N_{Gz} < 1000$, $80 < N_{Re} < 6000$, and $20 < N_{Pr} < 100$. It can predict the data of Berg and Bonilla [13] satisfactorily at higher Graetz numbers and that of Seban and McLaughlin at lower Graetz numbers. At very low Prandtl numbers, it predicts higher values.

Momentum and heat transfer analogy

The results of momentum and heat transfer for coiled pipes have been analysed using Prandtl's analogy equation [20]. It can be found from Fig. 14 that the points deviate in the order of the curvature ratios of coiled pipes. Hence the modification of analogy equation is essential. The free vortex flow occurring in coiled pipes introduces disproportionate influence on both fluid friction and heat transfer. The modified analogy equation is,

$$\{1 + [0{\cdot}5941 - 5{\cdot}3(D_i/D_c)][2{\cdot}303 \log (t_w - t_1)/(t_w - t_2)]\}$$

$$= \frac{2f_cL}{D_i}\frac{1}{1 + 5\sqrt{(f_c/2)}(N_{Pr} - 1)}$$

wherein the slopes and intercepts are correlated with the curvature ratio of helical coil or average curvature ratio of spiral tube coils. The modified equation can predict the rates of heat transfer from the rates of momentum transfer satisfactorily for coiled pipes.

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Résumé—Les donées expérimentales sur la perte de charge et le transport de chaleur pour un écoulement de solutions aqueuses de glycérol dans différents types de tuyaux hélicoïdaux et en spirale sont présentés dans le cas de l'écoulement laminaire lorsque N_{Re} varie de 80 à 6000. Une corrélation empirique est établie qui peut rendre compte des données actuelles aussi bien que de celles disponibles dans la littérature avec une dispersion de $\pm 10\%$. L'équation classique de l'analogie entre la quantité de mouvement et le transport de chaleur est utilisée pour analyser les données actuelles.

Zusammenfassung—Für Laminarströmung in Bereich von $N_{Re} = 80$–6000 in verschiedenen Arten von Spiralrohren werden Daten für Druckabfall und Wärmeübergang für wässerige Glycerollösungen angegeben. Eine empirische Korrelation fasst sowohl die hier enthaltenen Daten als auch die in der Literatur angegebenen bis auf $\pm 10\%$ Abweichung zusammen. Zur Analyse der Daten sind die konventionellen Gleichungen der Analogie von Impuls und Wärmeübergang herangezogen.

Аннотация—Представлены данные по перепаду давления и теплообмену при ламинарном течении водяного раствора глицерина в различных видах спиральных трубок в области чисел N_{Re} от 80 до 6000. Найдена эмпирическая зависмость, обобщающая результаты не только этой работы, но и другие литературные данные с точностью $\pm 10\%$. Для анализа этого исследования используется обычная аналогия теплообмена и количества движения.